User Guide

P3δ

Partial Pole Placement using Delay action

P3δ development team

Contents

Disclaimer

- When computing roots the software might turn on ”Not Responding”, do not close the software, it’s working.
- If you encounter any issue, or bug with the software, please contact us through our website in the ”Contact us” section.
- Issues might be encountered with the equation shown on the software. However, the ”Copy to clipboard” will copy the right LaTeX friendly equation.
- If in control oriented, a value of dominancy (s_0) is given but a delay (τ) sensitivity plot is required, please change the user input back to delay (τ) and insert the computed value of τ from the previous run where s_0 is given.

Overview

1 What is P3δ ?

Control systems often operate in the presence of delays, primarily due to the time taken to acquire the information needed for decision-making, to create control decisions and to execute these decisions.

Commonly, such a time-delay induces desynchronizing and/or destabilizing effects on the dynamics. However, some recent studies have emphasized that the delay may have a stabilizing effect in the control design. In particular, the closed-loop stability may be guaranteed precisely by the existence of the delay. Furthermore, a growing literature exhibits delayed-control-design in a wide range of applications; just to cite two examples :

- Control of flexible mechanical structures
- Regulation of networks

This project aims to associate efforts from an applied mathematics, control theory, mechanical engineering and computer science points of view to build an effective interface-based-design on recent methodologies and algorithms (symbolic/numerical) exploiting such delay-stabilizing properties.

Thereby, it is possible to draw the complete picture in the parameter space allowing to generate reduced-order-controllers.

A particular interest will be devoted to deeply investigate the system's parameter variations' effects on its qualitative properties; stability, oscillation, positivity. Unlike methods based on finite-spectrum-assignment, the targeted- controllers-design does not render the closed-loop-system finite dimensional, instead it controls its spectral abscissa.

The recently recovered intrinsic properties of time-delay systems motivate several new perspectives, which will be developed in this project. The purpose is fourfold:

1. Better improve the understanding of time-delay systems' spectrum distribution in the retarded as well as the neutral cases and, especially, determine the locus of the corresponding spectral abscissa
2. Establish a general/unified control approach for the spectral abscissa assignment based on the extension of recent results by part of the team
3. Achieve a friendly-user-interface consisting in an effective implementation of the generation of delayed controllers. Despite the existence of several algorithms and implementations for characterizing the spectrum of time-delay systems, none of them is able to handle control-oriented parametric systems
4. Test the efficiency of the targeted control method, tools and interface through the vibration quenching in mechanical and robotic structures.

2 Multiplicity Induced Dominancy

On a physical system, adding a delay induced a unstable state on the system instead of stabilizing it.

The Multiplicity-Induced-Dominancy (MID) is a mathematical method applied on those physical system used to find a way to stabilize the system, by including this delay.

To do that, the system need to be expressed in the Laplace form :

$$Q(s, \tau) = P_0(s) + P_1(s)e^{-\tau s} \tag{1}$$

With :

- P_0 : The non delayed polynomial describing the system
- P_1 : The delayed polynomial describing the delayed command/gain
- τ : The delay

Then with the derivatives of the system, you find the remaining data and finally solve the system in a precise space.

The main idea is to take advantage of the delay to generate a stabilizing command through root multiplicity and dominancy.

More details can be found in our references mentionned later on.

Quickstart

In P3δ, we are working with the following quasi-polynomial :

$$\sum_{i=0}^{n} a_i s^i + \sum_{i=0}^{m} b_i s^i e^{-s\tau} \tag{2}$$

With :

- n : Degree of non delayed polynomial
- m : Degree of delayed polynomial
- a_i : System values
- b_i : Command values
- τ : Delay

The main idea is to find values (either command or both command and system) in order to place the dominant root at s_0.

1 MID analysis

In the analysis case, we give $n,\ m,\ \tau$ and s_0 as input values. After also giving the contour data, the software will return a plot verifying that the required dominancy has been achieved aswell as a LaTeX formatted equation exportable via the copy to clipboard button.

1.1 Steps

1. Start by entering n and m in the required fields and press enter on your keyboard to confirm
2. Choose "Classic MID (Generic)" in the combo box in the top middle frame
3. Enter s_0 the dominancy and the delay value τ
4. Enter contour values, it is a rectangle defined by $x_{min},\ x_{max},\ y_{min},\ y_{max}$.
5. Press the "Confirm" button

   The following steps are optional and should be done if you wish to obtain the time domain response of the controlled system.

6. Choose the initial condition type (constant / polynomial / exponential / trigonometric)
7. Enter the simulation time
8. Enter the requested values for the initial condition on $[-\tau,\ 0]$, please be aware that if **polynomial** is chosen, the degree will be requested and then the coefficients will be asked through the user interface.

9. **For exponential, trigonometric and constant** : Press enter on your keyboard after completing the values to generate the figure

10. **For polynomial** : Press the confirm button in the initial condition frame

1.2 Output

- Root spectrum plot,
- Quasi-polynomial equation exportable in LaTeX on the middle right frame,

1.3 Time domain Simulation

By adding a type of function (Constant, Polynomial, Exponential or Trigonometric) from $[-\tau,\ 0]$, and a time T, the software will plot the response in Time of the system in [-τ, T].

If the dominancy roots is positive the response will diverge, else it converge to more or less 0 (depend of if the dominancies are complex values or not).

1.4 Recap

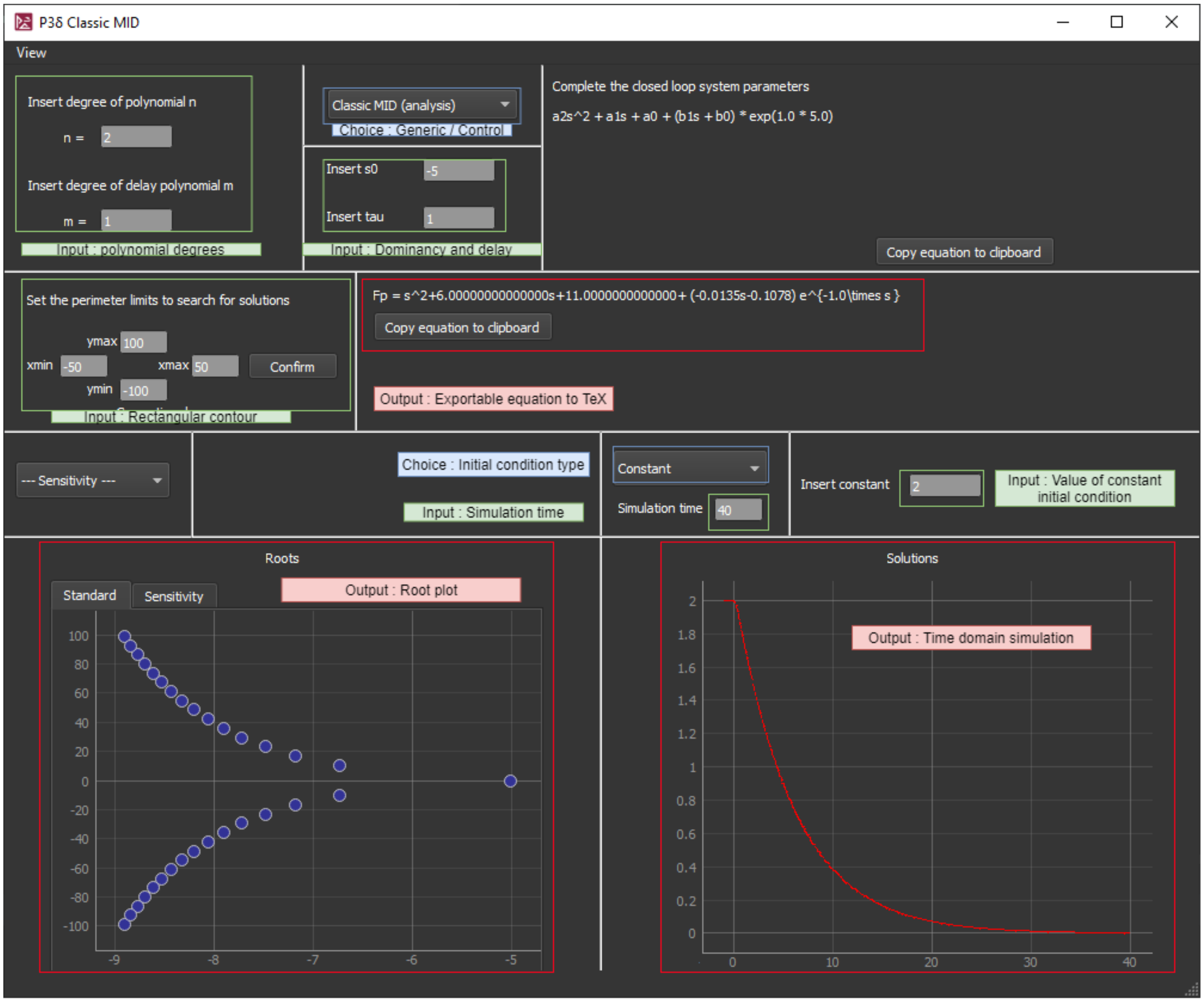

Figure 1: Generic MID using P3δ UI

2 Control oriented MID

The control oriented version of our software should be the one used by users if they already have a defined system and either a required dominancy value or delay.

The major difference with the analysis version is that here, the a_i values describing the system are given by the user and only one of the parameters (s_0 or τ) is given and the other one is computed with our algorithm.

2.1 Steps

1. Start by entering n and m in the required fields and press enter on your keyboard to confirm
2. Choose "Control Oriented MID" in the combo box in the top middle frame aswell as the parameter you wish to give as an input (either s_0 or τ)
3. Enter either the dominant root or the delayed depending on your previous choice
4. Enter the values of a_i, $i \in [\![0,\ n-1]\!]$, considering that $a_n = 1$
5. Press the "Confirm" button located in the same frame
6. Enter contour values, it is a rectangle defined by $x_{min},\ x_{max},\ y_{min},\ y_{max}$.
7. Press the "Confirm" button

   The following steps are <u>optional</u> and should be done if you wish to obtain the time domain response of the controlled system. Sensitivity will be explained further.
8. Choose the initial condition type (constant / polynomial / exponential / trigonometric)
9. Enter the simulation time
10. Enter the requested values for the initial condition on $[-\tau,\ 0]$, please be aware that if **polynomial** is chosen, the degree will be requested and then the coefficients will be asked through the user interface.
11. **For exponential, trigonometric and constant** : Press enter on your keyboard after completing the values to generate the figure
12. **For polynomial** : Press the confirm button in the initial condition frame

2.2 Outputs

- Gain values (b_i, also called α_i)
- Dominancy and delay where one of them has been computed
- Root spectrum plot
- Quasi-polynomial equation exportable in LaTeX on the middle right frame
- Sensitivity plot around the dominancy.

Please note that if maximal dominancy $(n+1)$ can not be reached by using the given parameters, the software will raise an exception and will not return the root spectrum although the label in the Contour frame states that the computing is finished.

The management of such situations will be handled in a further version by lowering the multiplicity of s_0.

Another important information is that maximal multiplicity does not imply dominancy of the root as in some exceptions, the software will give the appropriate and expected values and plot so the user should find more appropriate values of requested delay as this only happens when the given parameter is τ.

This should not happen when the dominancy value is given and the delay value, computed.

2.3 Time domain Simulation

By adding a type of function (Constant, Polynomial, Exponential or Trigonometric) from $[-\tau,\ 0]$, and a time T, the software will plot the response in Time of the system in [-τ, T].

If the dominancy roots is positive the response will diverge, else it converge to more or less 0 (depend of if the dominancies are complexes or not).

2.4 User giving dominancy value

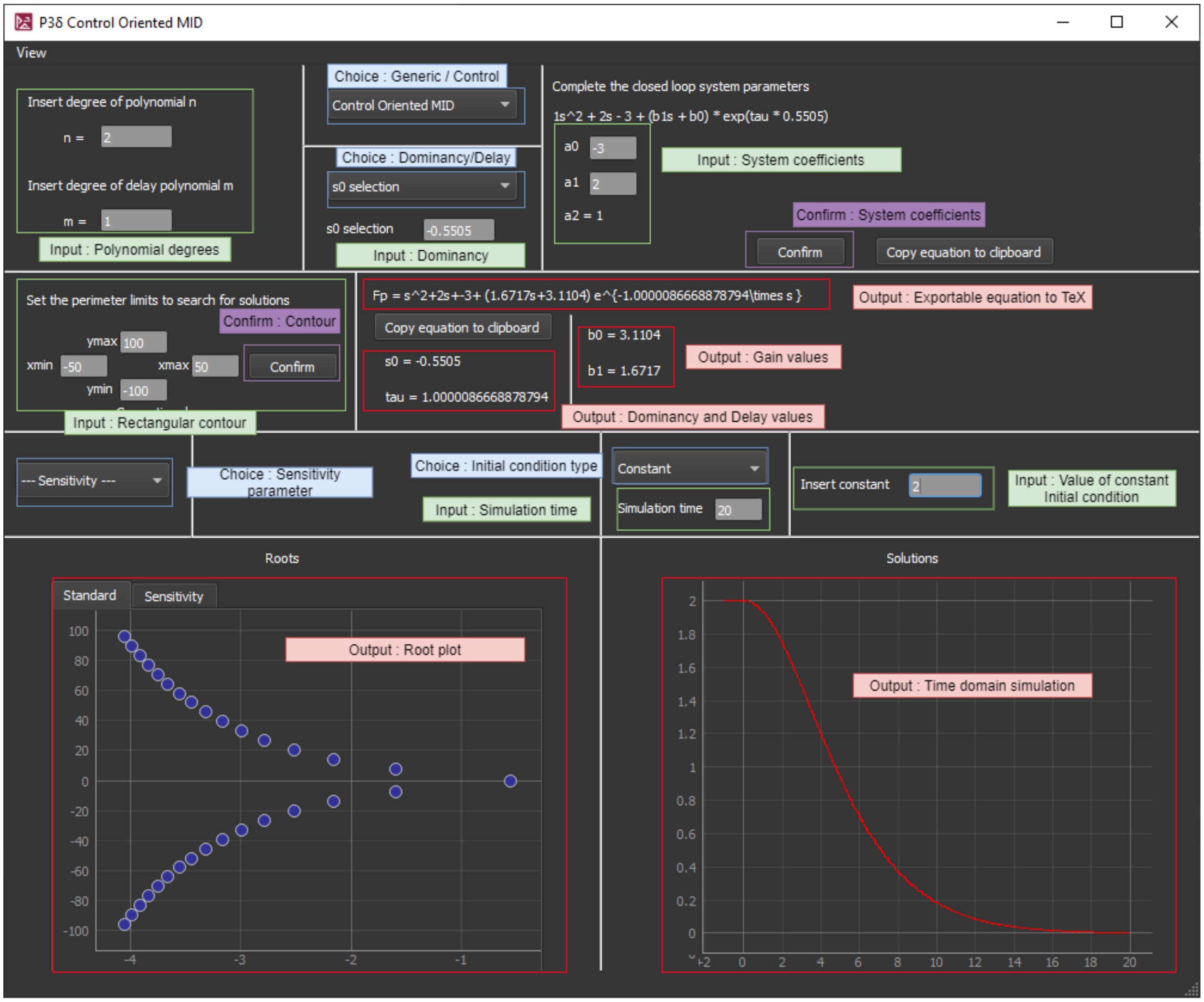

Figure 2: Control oriented MID using P3δ UI - Giving s_0

2.5 User giving delay value

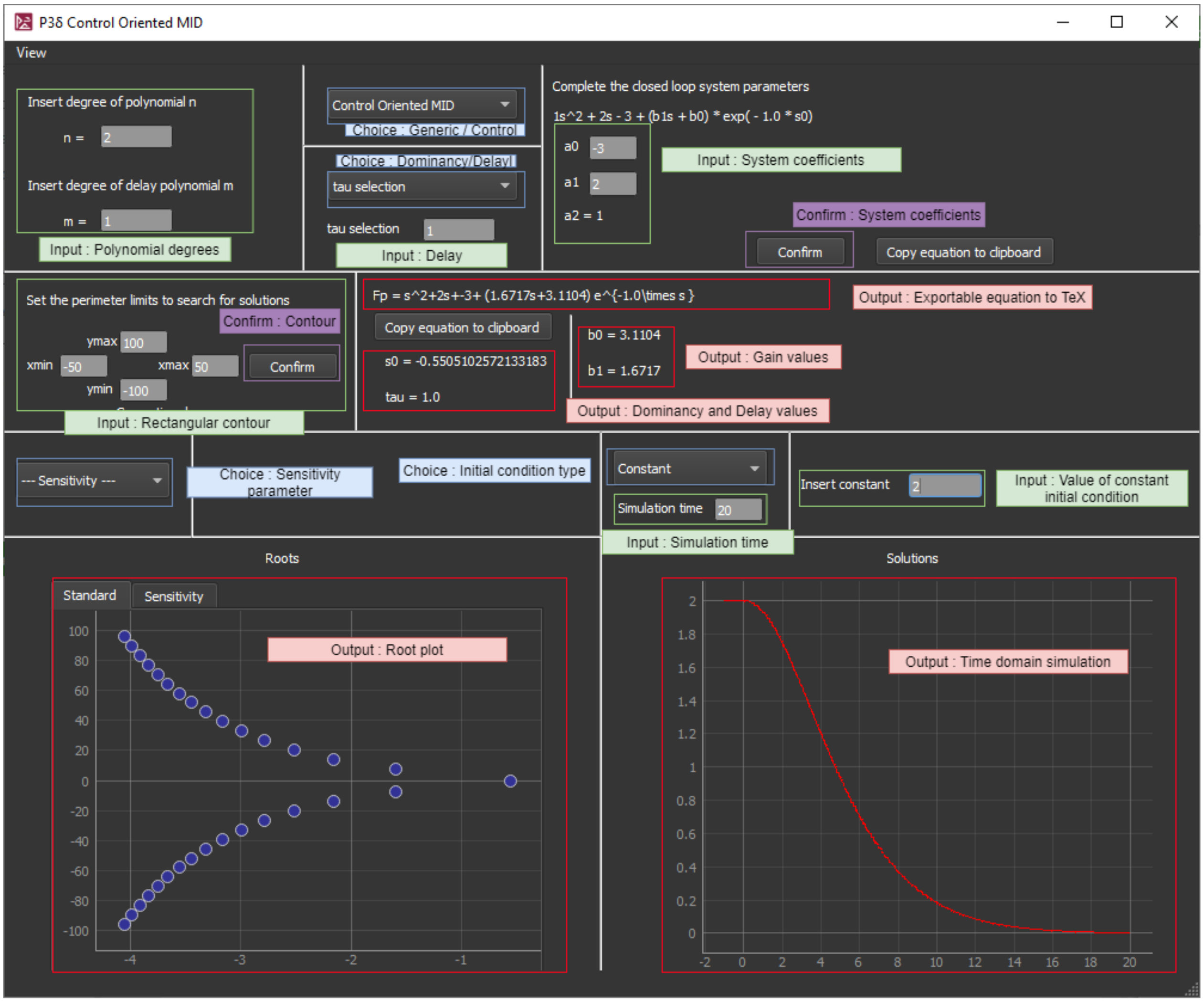

Figure 3: Control oriented MID using P3δ UI - Giving τ

2.6 Delay sensitivity

When the user gives a nominal value of delay, this value can sometimes vary meaning that it will exist in a given $[\tau - \varepsilon,\ \tau + \varepsilon]$ interval around the first given value.

With P3δ, it is possible to see the response and behaviour of the roots when the delay is uncertain and exists in an interval. In order to do so, the previous steps should first be done until the roots and the corresponding plots are obtained. Then, the steps to get the sensitivity plot are the following :

- Select the sensitivity tab in the "Roots" plot
- Select tau sensitivity in the combo box above the "Roots" plot
- Enter the step value ε
- Enter the number of iterations N

- Choose the new solving contour centered around the dominant root using the contour frame that was previously used to compute the roots
- Press confirm in the contour frame

The existence interval of the delay will thus be $[\tau - N\varepsilon,\ \tau + N\varepsilon]$.

Figure 4: Control oriented MID using P3δ UI - Sensitivity over τ

Values of $\tau^* \in [\tau - N\varepsilon,\ \tau[$ are shown on the plot in blue-like colors and the values of $\tau^* \in\ ]\tau,\ \tau + N\varepsilon]$ are shown in right-like colors. The black diamond symbols correspond to the roots found for the nominal value of τ.

As we can see this uncertainty over the delay values creates a shift of the roots and especially of the highest multiplicity one which is the one we are interested in.

Features

1 Implemented features

As of June 1^{st} 2020 with the first version of P3δ, the features are the following :

- Analysis mode to illustrate and get a better understand of the MID method for pole placement using delay action,
- Control oriented mode to allow users to apply MID to their own systems given a set of constraints and parameters,
- Spectrum of roots for both Analysis and Control oriented,
- Exportable and LaTeX friendly equation for both modes,
- Interactive plots,
- Switching between *Dark* and *Light* modes,
- Simulation in time domain for various types of functions (constant, polynomial, exponential and trigonometric) as initial condition on $[-\tau,\ 0]$,
- Sensitivity over one of the parameters (τ) with an interval of existence for its existence in Control oriented
- Software reset/reboot
- Showing constraint polynomial over s and τ shall given values not be in a correct interval

2 Planned features

- Sensitivity over the system coefficients (a_i)
- Multithreading abilities to avoid fake freeze issue
- Performance comparison with other control methods
- Non maximal dominancy management for control oriented
- Larger choice of command type
- Other pole placement approaches

Illustrative examples

In order to illustrate the use of our software. We will expose some illustrative examples. The first example is taken from [1].

1 Exponential decay

As an example, let's take the following differential equation.

$$\dot{\xi}(t) + a_0\,\xi(t) + a_1\,\xi(t-\tau) = 0, \tag{3}$$

After the Laplace transformation,

$$Q(s,\tau) = s + a_0 + a_1 e^{-s\tau} \tag{4}$$

From that we observe

$$Q(s,\tau) = P_0(s) + P_1(s)e^{-s\tau} \tag{5}$$

Where :

- $P_0(s) = s + a_0$ the non-delayed polynomial with a degree $n = 1$
- $P_1(s) = a_1$ the delayed polynomial with a degree $m = 0$
- τ is the delay.

For a given positive delay, equation (4) admits a double spectral value at $s = s_0$ if, and only if,

$$s_0 = -\frac{a_0\tau + 1}{\tau} \text{ and } a_1 = \frac{\mathrm{e}^{s_0\,\tau}}{\tau}. \tag{6}$$

1.1 Using analysis

When using analysis, we have to provide both s_0 and τ. In order to get both values, knowing that we have the information over τ, we can use the following graph :

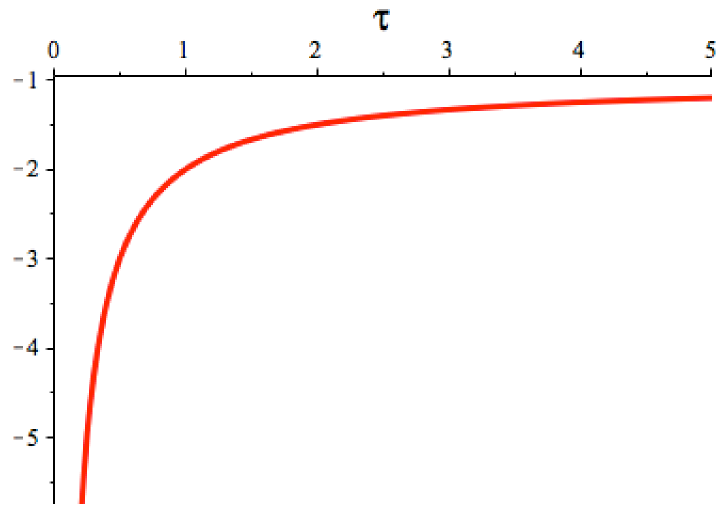

Figure 5: s_0 to τ connection

Using this graph, if we consider a delay value of $\tau = 1$, we can see that s_0 is about -2. We can then use our software to check whether our required dominancy is guaranteed with such a configuration :

The rightmost root s_0 corresponding to Eq.(4), where system (6) is satisfied, varies in the interval $s_0 \in]-\infty, -a_0[$. Fig.5 illustrates the behavior of the rightmost root with respect to the time-delay variation.

With a Time domain simulation, we suppose the response to be constant at 2 in $[-\tau; 0]$, and due to a negative dominancy roots, the response given by the software is converging to 0, and the system seems to be stable after 4s.

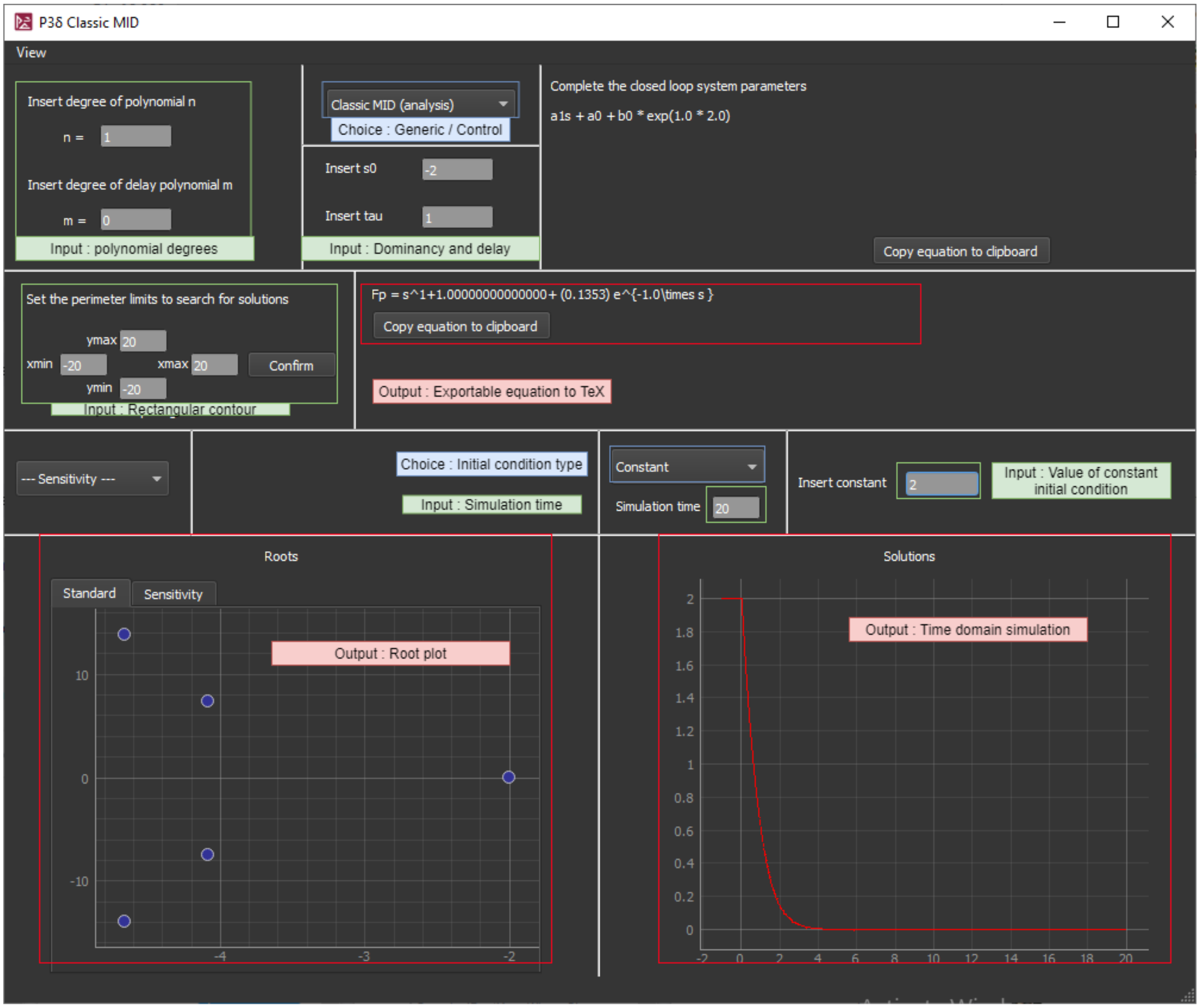

Figure 6: Exponential Decay - Using analysis

1.2 Using control oriented

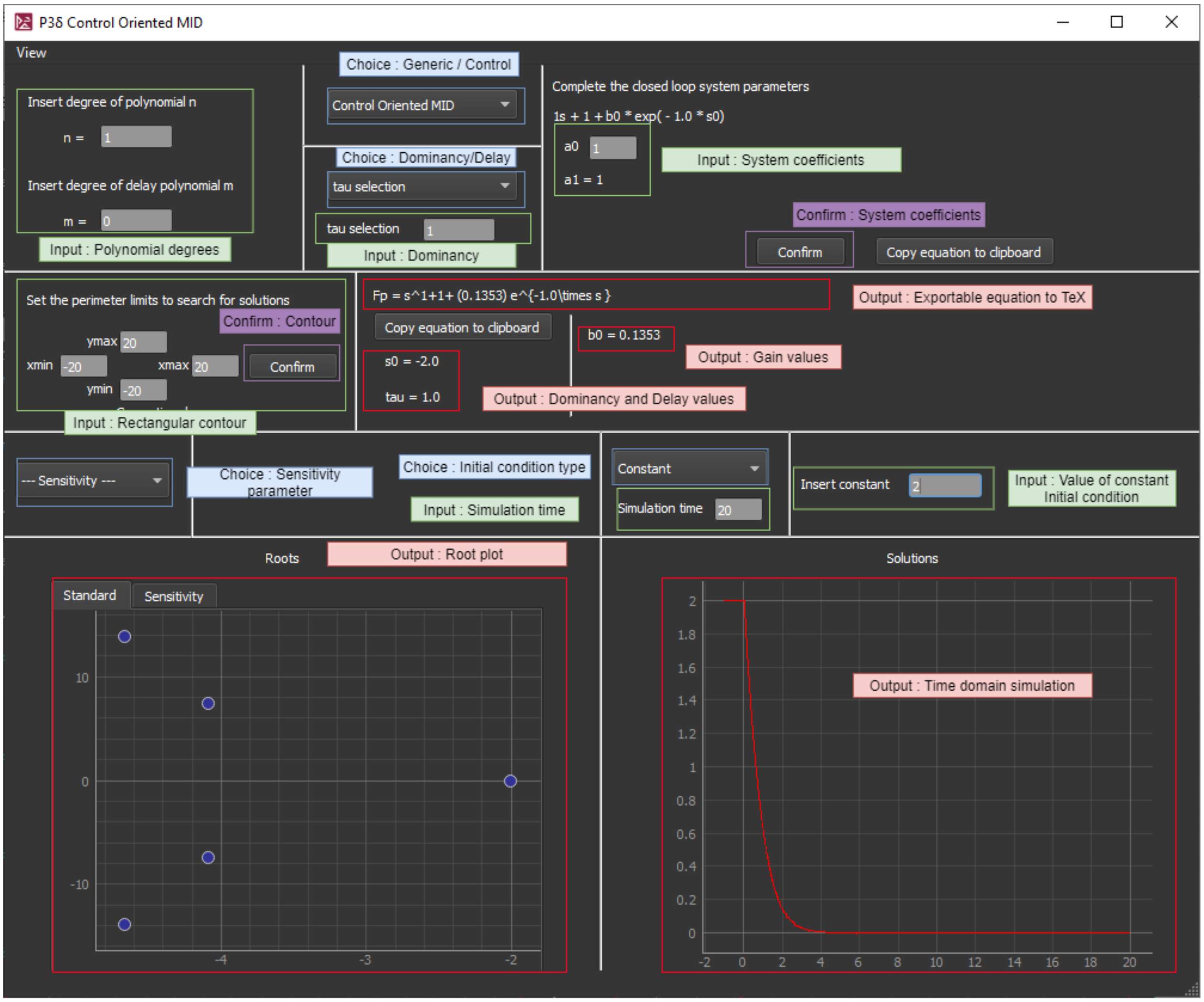

Figure 7: Exponential Decay - Using control oriented - Input : τ

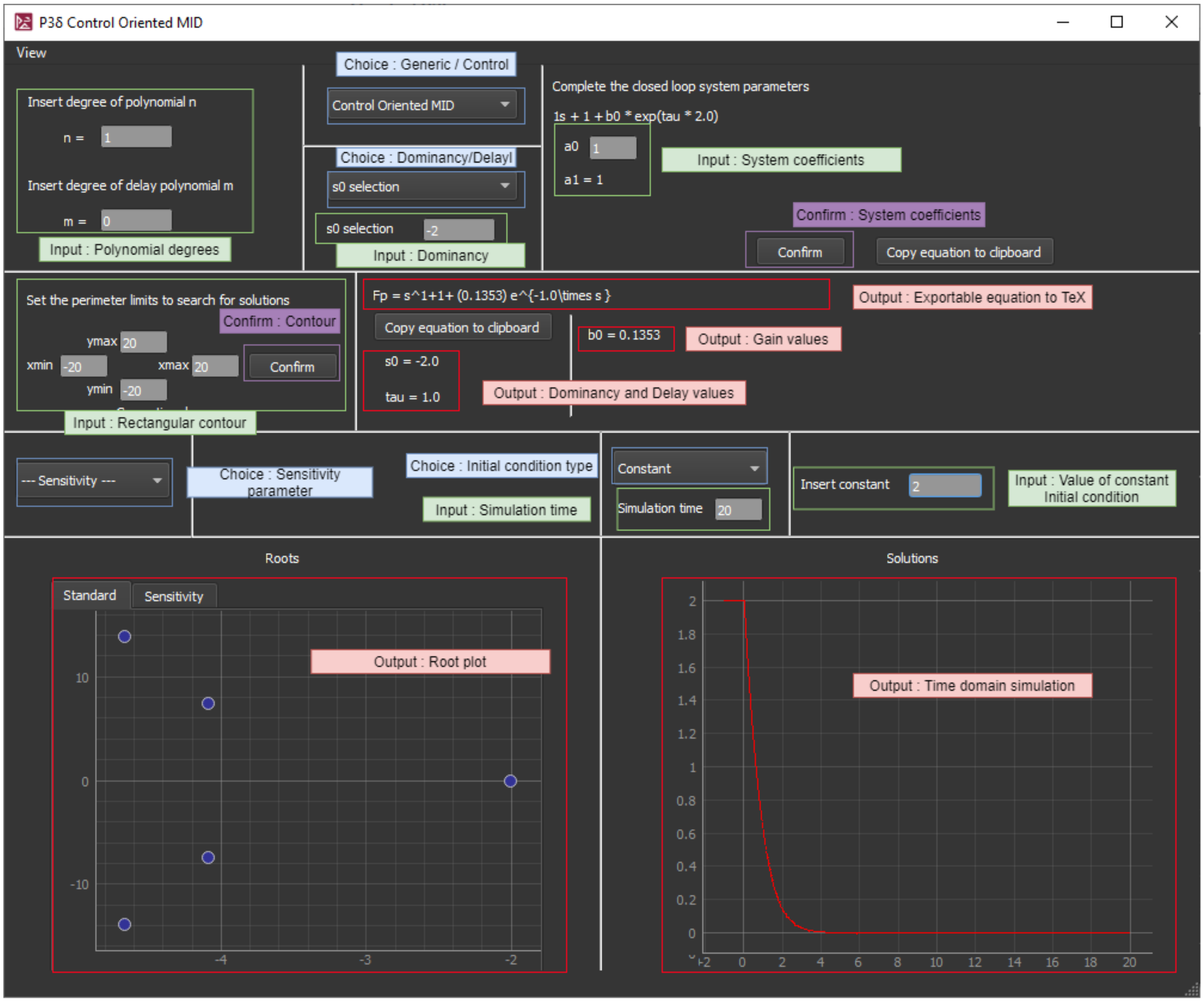

Figure 8: Exponential Decay - Using control oriented - Input : s_0

2 Stabilizing the Double Integrator via a Delayed-Feedback controller

Conforming with the presented approach, we consider the control problem $\ddot{\xi}(t) = u(t)$ where the controller structure is given by $u(t) = -\alpha_0\,\xi(t-\tau) - \alpha_1\,\dot{\xi}(t-\tau)$. So that, the characteristic equation corresponding to the closed-loop system is:

$$Q(s,\tau) = s^2 + (\alpha_1\, s + \alpha_0)\; e^{-\tau\, s} = 0.$$

Note that the inclusion of the gain $\alpha_1 \neq 0$ in the control structure increases the degree of the resulting quasipolynomial to 4. Applying the result of Theorem 4.2 from Section 4 , we assign a dominant negative spectral value with multiplicity 3 at $s_+ = \frac{-2+\sqrt{2}}{\tau}$ (corresponding to the dominant decay rate of the solution) when α_0 and α_1 are chosen such that:

$$\alpha_0 = 2\,\frac{\left(-7+5\,\sqrt{2}\right)\mathrm{e}^{-2+\sqrt{2}}}{\tau^2} \quad \text{and} \quad \alpha_1 = 2\,\frac{\left(\sqrt{2}-1\right)\mathrm{e}^{-2+\sqrt{2}}}{\tau}.$$

Observe that decreasing the delay value improves the exponential decay rate of the time-domain solution. For a $\tau = 1$

$$s_+ = -0.585786438, \quad \alpha_0 = 0.07912233780 \quad \text{and} \quad \alpha_1 = 0.4611587914$$

Using P3δ, we find the following results :

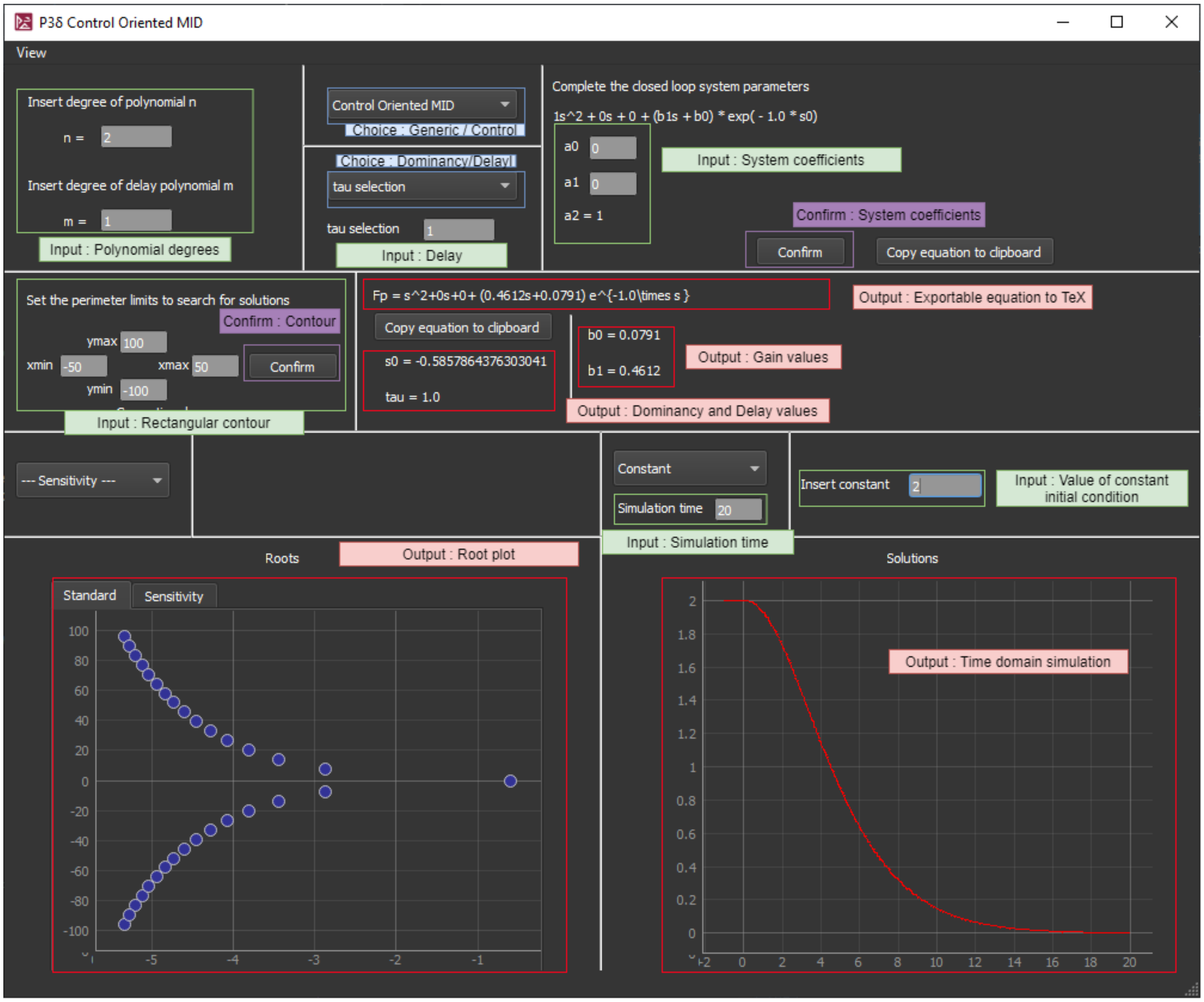

Figure 9: Double Integrator with delayed-feedback controller - Using control oriented - Input : τ

If we are uncertain about the delay value, we can use the sensitivity feature to get an idea of the roots and thus the system's behaviour by considering an interval of existance for τ. Here, let's consider the following

- Step value : $\varepsilon = 0.005$
- Number of iterations : 6

Thus, the interval is $[\tau - 6 \times 0.005,\ \tau + 6 \times 0.005]$ which translates to $[0.97,\ 1.03]$ with a step of $\varepsilon = 0.005$. We get the following response sensitivity plot from our software :

Figure 10: Double Integrator with delayed-feedback controller - Using control oriented - Sensitivity over τ

References

Underlined titles are clickable and will redirect you to the source documents.

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List of Figures